

17. State and prove Sequence Lemma.
 18. Prove that the components of X are connected disjoint subspaces of X whose union is X , such that each nonempty connected subspace of X intersects only one of them.
 19. Let X be locally compact Hausdorff. Let A be a subspace of X . Prove that if A is closed in X or open in X then A is locally compact.
 20. Prove that a subspace of a completely regular space is completely regular and product of completely regular.
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NOVEMBER/DECEMBER 2025

GMA32/DMA32 — TOPOLOGY

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define topology space.
2. Define Housdorff space.
3. Define metric topology.
4. Define box topology.
5. State intermediate value theorem.
6. Define Linear Continuum.
7. State Lebesgue number lemma.
8. Define Compactification.
9. When a space is said to be second countable?
10. State urysohn metrization theorem.



SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) If A is a subspace of X and B is a subspace of Y then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

Or

- (b) Let Y be a subspace of X . Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y .

12. (a) Prove that the uniform topology on $\mathcal{R}^J$ is finer than the product topology and coarser than the box topology, where J is infinite.

Or

- (b) Let $f: X \rightarrow Y$ be continuous then prove that for every convergent sequence $x_n \rightarrow x$ in X , the sequence $f(x_n)$ converges to $f(x)$.

13. (a) Prove that the image of a connected space under a continuous map is connected.

Or

- (b) Prove that if X is a topological space, each path component of X lies in a component of X . And if X is locally path connected then the components and the path components of X are the same.

14. (a) Prove that a subspace A of $\mathcal{R}^h$ is compact if and only if it is closed and bounded in the euclidean metric d or the square metric ρ .

Or

- (b) Prove that compactness implies limit point compactness. Is the converse true?

15. (a) Prove that a subspace of a Hausdorff space is Hausdorff and a product of Hausdorff spaces is Hausdorff.

Or

- (b) State and Urysohn Lemma.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let A be a subset of the topological space X . Then prove that following :

- (a) $x \in \bar{A}$ if and only if every open set U containing x intersects A .

- (b) Supposing the topology of X is given by a basis then $x \in \bar{A}$ if and only if every basis element B containing x intersects A .